

with—on the outskirts of London in places where each could be furnished with at least one hundred acres of land at a total cost of no more than is asked for the 11½ acres now proffered.

The advantages of such outlying centres would be: (1) The students could live in the neighbourhood of the institution, either at home or in hostels, and would not be compelled to take a long journey twice a day. (2) There would be abundance of room for all purposes, including recreation. (3) Each subject would be able to have its own area of ground for the erection of a suitable institute, and for permitting future extension. (4) Temporary buildings might be put up until experience has shown what character of permanent buildings ought to be erected. (5) The classes would not be of an unwieldy size; for we might assume for each of the four centres two hundred and fifty students, instead of the one thousand assumed for the central institute, *i.e.* for each practical subject.

American experience has shown that one hundred acres is not too much land to provide for the buildings of a modern university, and a scheme which assumes that a single university for a city the size of London can be accommodated in a space of ten, or even twenty, acres is self-condemned at the outset.

E. SHARPEY SCHAFFER.

University New Buildings, Edinburgh, June 7.

High Rates of Ascent of Pilot-Balloons.

ABNORMAL rates of ascent shown by pilot-balloons have in recent years aroused considerable interest amongst aerologists. These digressions were mostly ascribed to the occurrence of vertical air-currents, but three years ago Wenger (*Annalen der Hydrographie*, 1917, Hamburg) suggested¹ that, for the greater part, the observations did not permit of such an explanation. He advanced the theory that these abnormal rates were chiefly caused by increased turbulence of the air, and he stated, making use of the Lindenberg material, that the rates observed depended on different conditions of the air, as wind, friction, etc., which, no doubt, must have a notable influence on its turbulence.

In the same journal, however, it was shown that a large number of observations made at Sofia (Bulgaria) gave strong evidence that actually large vertical air-currents had occurred, and that the high and low rate should, without doubt, be ascribed to upward and downward movements of the air.

Between 1912 and 1917 much material regarding these rates was gathered by the Batavia Observatory, and peculiar circumstances make this material of critical value, for it consists of three series taken in three localities differing in character, the ascents being made at various hours during the day and night.

The first series was taken at Batavia during the dry, sunny season, when land- and sea-breezes are developed strongly; and the second at Bandung, a town situated on a plateau 700 m. above sea-level, surrounded by mountains. Insolation in the latter case did not differ much from that at Batavia; the mountain- and valley-breezes were only slight.

The third series was taken by Dr. Boerema on a small coral islet in the Java Sea at the end of the west monsoon. There the influence of insolation and of land- and sea-breezes is practically nothing.

Most of the balloons were observed from two points; also, up to 1 or 1½ km., half-minute readings were taken between the usual observations made every minute. Balloons of different weight and pattern were used; those for the night carried a second balloon

filled with acetylene and a burner. For each kind I calculated the average rate observed at the level of 3 or 4 km., and derived from it the rate for the layers beneath by applying the formula

$$v = ad - \frac{1}{2} (d = \text{air density}).$$

For the light balloons there remained, of course, a change of rate with height, which, however, I was unable to calculate, but I surmise was small.

The accompanying diagram (Fig. 1) gives for the lowest layers up to 3 km. the departures of the mean rates observed from those at 3 or 4 km. Evidently it displays the contrast between the land and the sea influence, *i.e.* on land the rates are increased by day, but are normal by night, while at sea there is no distinct increase in the average rate.

At Batavia the positive digression runs parallel with the strength of the sea-breeze and with the change of

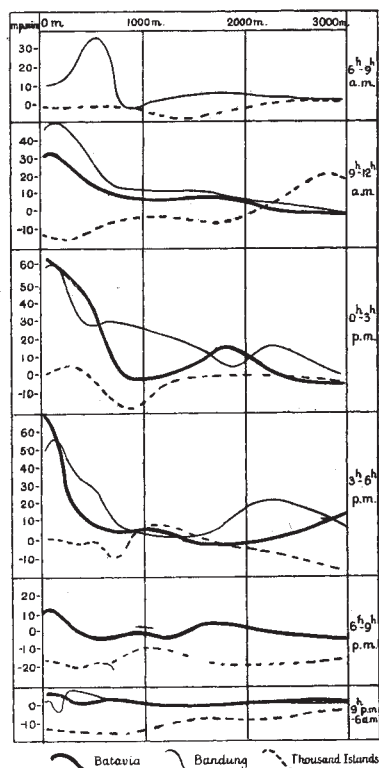


FIG. 1.

wind velocity caused by the Espy-Köppen effect. At Bandung also this parallelism seems to exist.

Consequently, at first sight, the explanation proposed by Wenger might be given: During the day turbulence is enhanced on land by increase of wind velocity, by friction with the surface, and by insolation; at sea, on the contrary, these causes are not present. However, on more detailed examination this explanation is not confirmed. Thus the rate of ascent did not prove to depend on wind velocity, as the following figures clearly show:

Batavia.							
9 a.m.-noon		Noon-3 p.m.		3-6 p.m.			
Δ Rate of ascent m. p. min.	Wind velocity m. p. sec.	Δ Rate of ascent m. p. min.	Wind velocity m. p. sec.	Δ Rate of ascent m. p. min.	Wind velocity m. p. sec.	Δ Rate of ascent m. p. min.	Wind velocity m. p. sec.
3	3.8	-4	8.7	4	8.0	—	—
63	4.0	-64	7.5	56	5.6	—	—
—	—	136	6.4	133	6.9	—	—

¹ It came under my notice only a few months ago.

Bandung.	
9 a.m.-6 p.m.	
Δ Rate of ascent m. p. min.	Wind velocity m. p. sec.
10	2.9
39	3.7
103	2.3

Also, the wind velocity scarcely ever reached the values (>15 m. per sec.) at which, according to Wenger, the influence of wind velocity begins to increase turbulence, so that a notable increase in the rate of ascent is to be expected.

Moreover, insolation is strongest between 9 a.m. and noon; afterwards, clouds mostly weaken it or prevent further increase. On the other hand, the rate of ascent at Batavia between noon and 3 p.m. considerably exceeds that between 9 a.m. and noon.

Thus an explanation of the observed rates by Wenger's theory practically fails; on the contrary, the supposition of vertical air movement is tenable.

For some years I supposed that the air had to rise in columns, and, the surrounding air being sucked in, the balloon in most cases would also be sucked in, and afterwards would not leave the ascending air-columns. Later I read that J. S. Dines was inclined to this conception. The criticism of this view offered by Wenger must be accepted; but why should not both causes co-operate in the lowest strata?

Indeed, I have found that my results and those of Wenger coincide regarding the change of rate of ascent when, passing upwards, wind velocity varies.

Denoting this change by $10^2 \frac{dv}{dz}$ (v =wind velocity in m. per sec., and z =height in m.), I found:

Batavia.			Thousand Islands.		
$10^2 \frac{dv}{dz}$	Δ Rate of ascent m. p. sec.	No. of cases	$10^2 \frac{dv}{dz}$	Δ Rate of ascent m. p. sec.	No. of cases
1.2	0.7	22	0.7	1.3	28
-1.1	-1.0	42	-0.4	-1.1	38
-2.2	-0.6	20			

As these values show, the change of rate with wind velocity is not developed strongly, the percentage of cases in which dv/dz and Δ rate were of the same sign being respectively for Batavia and the Thousand Islands 63 and 68 only.

Finally, we are obliged to accept the view that in the sea-breeze the air must rise as the breeze dies out at a moderate distance from the coast. Also, the air seems to rise no higher than the sea-breeze itself, the rate of rise diminishing with its horizontal velocity.

Moreover, I think the material collected on and near Java is not favourable to the idea of such a preponderating influence of turbulence as Wenger accepts; on the contrary, it corroborates the assumption of ascending columns.

The formation of the fine-weather cumuli, to be observed every sunny day in the tropics, is clear evidence of the general occurrence of these ascending air columns.

W. VAN BEMMELN.

On board s.s. *Tjisondari*, Pacific.

A New Method for Approximate Evaluation of Definite Integrals between Finite Limits.

GAUSS, I believe, gave a very large number of forms for approximate evaluation of definite integrals between finite limits. His formulæ are all based, like Tchebycheff's rules, on the assumption that the integrand is expressible approximately by a finite number of terms of the series $a+bx+cx^2+dx^3+\dots$. His plan was to use a minimum number of suitably

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weighted ordinates to give him the exact value of the integral for a specified number of terms.

Taking the range of integration to be from -1 to $+1$, which can be done without any loss of generality, his simplest integral is

$$\int_{-1}^{+1} f(x) dx = f(-x_1) + f(x_1),$$

where

$$x_1 = \frac{1}{\sqrt{3}}.$$

This formula with two ordinates gives *exact* values for

$$\int_{-1}^{+1} (a+bx+cx^2+dx^3) dx,$$

and is in that respect on a par with Simpson's formula, which has three ordinates weighted in the proportions 1, 4, 1, and situated at the ends and middle of the range.

The next Gauss formula is

$$\int_{-1}^{+1} f(x) dx = \frac{1}{3} \{ 5f(-x_1) + 8f(0) + 5f(x_1) \},$$

where

$$x_1 = \sqrt{\frac{3}{5}}.$$

This is exact up to and including the term in x^5 in the series. Put in the same form as Mr. Merchant's formula (which is also exact up to the x^3 term) in NATURE of June 3, it becomes

$$\int_0^1 f(x) dx = \frac{1}{8} \{ 5f(x_1) + 8f(x_2) + 5f(x_3) \},$$

where

$$x_1 = 0.1127, \quad x_2 = 0.5, \quad x_3 = 0.8873.$$

The third formula is

$$\int_{-1}^{+1} f(x) dx = \frac{1}{(A+B)} [Af(-x_1) + Bf(-x_2) + Bf(x_2) + Af(x_1)]$$

where

$$A = \frac{1}{2} - \frac{1}{4}\sqrt{\frac{5}{3}}, \quad B = \frac{1}{2} + \frac{1}{4}\sqrt{\frac{5}{3}},$$

$$x_1^2 = \frac{3}{5} + \frac{2}{5}\sqrt{\frac{5}{3}}, \quad x_2^2 = \frac{3}{5} - \frac{2}{5}\sqrt{\frac{5}{3}}.$$

Transferred to the other form, this becomes

$$\int_0^1 f(x) dx = 0.1739f(x_1) + 0.3261f(x_2) + 0.3261f(x_3) + 0.1739f(x_4)$$

where

$$x_1 = 0.0694, \quad x_2 = 0.3300, \quad x_3 = 0.6700, \quad x_4 = 0.9306.$$

This formula is exact up to and including the term in x^7 .

It may be noticed that although the weight factors are now incommensurate, they can be written with a very close degree of approximation as $\frac{1}{8}$ and $\frac{3}{8}$, and the integral then takes the form

$$\int_0^1 f(x) dx = \frac{1}{8} [8f(x_1) + 15f(x_2) + 15f(x_3) + 8f(x_4)].$$

Possibly Mr. Merchant might find that this form would be useful in ship design. The positions of the ordinates is not sufficiently close to even tenths to permit of such further modifications being made, but if the ship's half-length were divided into fifteen sections, the ordinates would come very near the first, fifth, tenth, and fourteenth. Some of the higher Gauss integrals might be found to fit in even more conveniently.

THOS. Y. BAKER.

Admiralty Compass Observatory, Slough,
Bucks, June 10.

The Royal Military Academy.

SIR GEORGE GREENHILL in an article in NATURE of April 29, entitled "Artillery Science," passes severe strictures on the Royal Military Academy—"The Shop." These reflect on the whole staff, especially the military staff, and as the officers are not permitted